Combinatorial optimization for undergraduate students

Lecture note 13. Matching-Edmond's blossom algorithm

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In this section, G is connected with a given matching M. We construct an alternating tree T with root r. Edges xy closing a cycle of even length will be ignored. Whenever we encounter an edge xy closing a blossom B, we stop the construction of T and shrink the blossom B.

Formally, the operation of contraction G/B a subgraph B is defined as follows:

- The vertex set is $(V(G) \setminus B) \cup \{b\}$.
- The edge set is obtained by removing all edges $uv \in E$ with $u \in B$ or $v \in B$, and adding an edge ub for all those $u \in V(G) \setminus B$ having a neighbor in B.

The new vertex b is called a *pseudovertex* of G/B.

Example)

The algorithm of Edmonds generalizes the method used in Example. Before describing, we ought to show that the shrinking process always works correctly. It will suffice to show that G/B contains an augmenting path if and only if G contains an augmenting path.

Lemma 1. *If G has an augmenting path starting at r, then G' contains an augmenting path with respect to the induced matching M/B starting at r.*

Continue)

Lemma 2. *If G' contains an augmenting path with respect to M/B starting at r, then G contains an augmenting path.*

Algorithm of Edmonds :