Combinatorics and graph theory

Lecture note 10. Coloring maps.

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Consider a political map showing the boundaries of nations.

We consider two regions neighbors if they have at least a small piece of border in common, so it is not sufficient if the boundaries touch in one or several points.

We want to color each nation by some color such that no two neighboring nations get the same color. What is the minimum number of colors?

There is a map such that three is not sufficient number of colors.

Theorem 1 (Appel and Haken, 1976)**.** *Any map can be colored by 4 colors.*

Known proofs of this fact are difficult and depend on analyzing a large number of cases by computer. So far, nobody has managed to do the proof by hand.

Robertson, Sanders, Seymour, Thomas (1995) simplied the original proof. Gonthier (2004) managed to create a proof fully verifiable by computer.

However, 5-color is easier.

Chromatic number

Let $G = (V, E)$ be a graph and $k \geq 0$ be an integer. A mapping $c : V \to \{1, 2, \ldots, k\}$ is called a coloring of G if $c(x) \neq c(y)$ for every edge xy. The *chromatic number* of G, denoted by $\chi(G)$, is the minimum k such that there exists a coloring $c : V(G) \to \{1, 2, \ldots, k\}$.

For proving about coloring of maps, it is sufficient to prove that we can color a planar graph with 5 colors, because of dual graphs.

Theorem 2. *Any planar graph G satisfies* $\chi(G) \leq 5$.

We give two proofs.

CONTINUE.