Combinatorics and graph theory

Lecture note 15. Probability - Applications

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Fall, 2018

Existence of large bipartite subgraphs

Theorem 1. *Let G be a graph with an even number $2n$ of vertices and with $m > 0$ edges. Then $V(G)$ can be partitioned into two n-element sets A and B such that more than $m/2$ edges go between A and B.*

Number of intersections of level at most k

We consider a set L of n lines in the plane so that no three lines of L meet at a common point, and no two of them are parallel. Let x be a point lying on none of the lines of L. An intersection point is a point where two lines cross. An intersection v has level k if the segment xv intersects exactly k more lines other than those two intersecting lines.

Theorem 2. *For any set of n lines, there exist at most $3(k+1)n$ intersections of level at most k.*

PROOF Continue

Average number of comparison
ALGORITHM QUICKSORT is a way of sorting.

Theorem 3. *Let $x_1 < x_2 < \cdots < x_n$ be a sequence of real numbers in an increasing order. Let π be a permutation of the set $\{1, 2, \ldots, n\}$, and let $T(\pi)$ be the number of comparisions made by QUICKSORT for the input $(x_{\pi(1)}, x_{\pi(2)}, \ldots, x_{\pi(n)})$. Then the expectation of $T(\pi)$ for a random permutation π is at most $2n \ln n$.*