Combinatorics and graph theory

Lecture note 17. Order from disorder - Ramsey type results

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There are several types of theorems similar to Ramsey's theorem.

Theorem 1 (Erdős-Szekeres lemma)**.** *An arbitrary sequence* $(x_1, x_2, \ldots, x_{k\ell+1})$ *of real numbers contains either a monotone increasing subsequence of length* $k+1$ *or a monotone decreasing subsequence of length* $\ell+1$.

PROOF

We consider points on a plane. A set of points is in general position if no three points lie on a line.

Theorem 2. *Any set of five points in the plane in general position has a subset of four points that form the vertices of a convex quadrilateral.*

Let $f(N)$ denote the minimum integer M for which any set of M points in general position must contain a convex N-gon. It is known that

- $f(3) = 3$
- $f(4) = 5$
- $f(5) = 9$
- $f(6) = 17$

The values of $f(n)$ for $n > 6$ are not known.

On the basis of these values, Erdős and Szekeres conjectured that $f(k) = 1 + 2^{k-2}$. They proved that by constructing examples, $f(k) \geq 1 + 2^{k+2}$.

Theorem 3 (Erdős- Szekeres theorem)**.** *For every $k \geq 3$, there exists $n(k)$ such that every $n(k)$ points on a plain in general position has a subset of k points that form the vertices of a convex k-gon.*

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